Application of a Non-Linear Viscoelastic Model to the Relaxation of Ligaments

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Introduction: The most successful model developed to describe viscoelasticity in soft tissues is the quasi-linear viscoelastic (QLV) model [1]. In the QLV model, the relaxation function is assumed to be a separable function of time and strain. This assumption requires that the rate of relaxation is constant for all levels of strain. Recent experimental evidence [2] demonstrated that the rate of relaxation is a non-linear function of strain, especially at small physiological strains. Therefore, a more robust model, which considers these strain-dependent nonlinearities, is necessary to characterize the relaxation behavior in soft tissues. A new transversely isotropic non-linear viscoelastic model for relaxation in ligaments is presented. In this model, a non-separable tensorial relaxation function that depends on the strain invariants and time is adopted. To the authors' knowledge, the model represents the first fully non-linear, finite strain, anisotropic viscoelastic model applied to ligaments.

Model Formulation: A single integral form of the Pipkin-Rogers model, considering tissue anisotropy and the dependence of rates of relaxation on strain, was adopted. The Cauchy stress, **T**, has the form [3]:

$$\mathbf{T}(t) = -p\mathbf{1} + \mathbf{F}(t)\left(\mathbf{R}[\mathbf{C}(t),0] + \int_0^t \frac{\partial \mathbf{R}[\mathbf{C}(\tau), t-\tau]}{\partial (t-\tau)} d\tau\right)\mathbf{F}(t)^T \quad (1)$$

where t is time, **F** is the deformation gradient, **C** is the right Cauchy-Green deformation tensor, p is the Lagrange's multiplier that accounts for incompressibility. The tensorial relaxation, $\mathbf{R}[\mathbf{C}(t), t]$, is assumed to be derivable from a scalar potential function defined as:

$$W = \tfrac{c_1}{2c_2} e^{c_2(I_4-1)^2}\left[a(I_4)\left(e^{-b(I_4)t} - 1\right) + 1\right] \quad (2)$$

where c_1 and c_2 are material constants that describe the elastic behavior, a = $a(I_4)$ and b = $b(I_4)$ are functions of I_4, which denotes the 4^{th} invariant of **C**, and describe the coupled dependency on strain and time.

Results: After assuming an isochoric axisymmetric deformation, the above model was curve-fit to experimental data on rabbit medial collateral ligaments published by Hingorani et al. [2] to find the values for c_1, c_2, a, and b. The values for c_1 and c_2 were determined to be c_1 = 132 MPa and c_2 = 154.7 using isochronal stress strain data. Stress relaxation data collected at two different strain levels were used to find the dependence of a and b on the strain. Figure 1 shows the stress relaxation data and the model fit on a log-log plot.

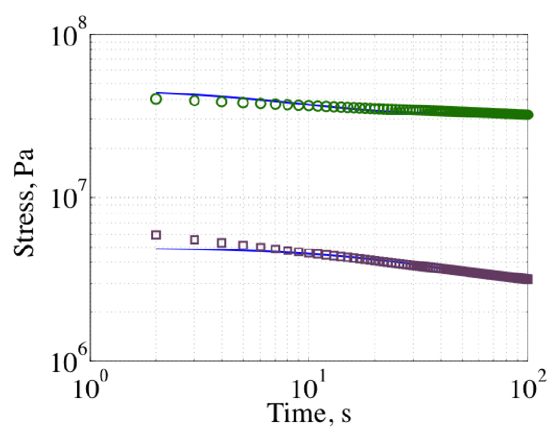

Figure 1: Model fit (blue line) and stress relaxation data collected at two strain levels: 0.81% (purple squares) and 3.57% (green circles) (Hingorani et al. [2]).

Conclusions: This work extends the general framework presented by Pipkin and Rogers [3] to ligamentous tissues. The model is formulated by considering the non-linear viscoelastic properties and anisotropy of these tissues. It represents a departure from the QLV model because it allows the rate of relaxation to depend on strain. Future studies will be conducted to extend the proposed model to describe other viscoelastic phenomena such as, for example, hysteresis and creep.

References

[1] Fung, Y.C. 1993. Biomechanics: Mechanical Properties of Living Tissues.
[2] Hingorani, R.V., et al., 2004. *Ann Biomed Eng*, **32** (2), pp. 306--312.
[3] Rajagopal, K.R. and Wineman, A.S., 2009. *Math Mech Solids*, **29** (10), pp. 490--501.