Proceedings of the ASME 2012 Summer Bioengineering Conference
SBC2012
June 20-23, 2012, Fajardo, Puerto Rico, USA

SBC2012-80050

A NONLINEAR VISCOELASTIC MODEL FOR THE RELAXATION BEHAVIOR OF TENDON

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INTRODUCTION

Tendons are viscoelastic materials which undergo stress relaxation when held at a constant strain. The most successful model used to describe the viscoelastic behavior of tendons is the quasi-linear viscoelastic (QLV) model [1]. In the QLV model, the relaxation function is assumed to be a separable function of time and strain. Recently, this assumption has been shown to be invalid for tendons [2] thus suggesting the need for new nonlinear viscoelastic models.

In this study, a transversely isotropic nonlinear viscoelastic model for the stress relaxation of rat tail tendon fascicles is presented. The model is formulated within the nonlinear viscoelastic framework set forth by Pipkin and Rogers [3] by considering recent theoretical developments by Rajagopal and Wineman [4] for anisotropic materials. The current model represents a departure from current viscoelastic models widely used in biomechanics for soft biological tissues because it incorporates a non-separable relaxation which is a function of the strain invariants and time. It is validated using stress relaxation data collected at multiple strain levels from rat tail tendon fascicles and compared to the predictions of the QLV model.

THEORETICAL DEVELOPMENT

The integral series proposed by Pipkin and Rogers [3] is used to describe the nonlinear viscoelastic response of the tendon fascicles. Only the first term of the integral series is considered, so that the first Piola-Kirchhoff stress tensor, $\mathbf{P}(t)$, at time t has the form

$$\mathbf{P}(t) = -p\mathbf{F}^{-\mathrm{T}}(t) + \mathbf{F}(t)\left(\mathbf{R}[\mathbf{C}(t),0] + \int_0^t \frac{\partial \mathbf{R}[\mathbf{C}(\tau),t-\tau]}{\partial(t-\tau)} d\tau\right) \quad (1)$$

where $\mathbf{F}$ is the deformation gradient tensor, $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$ is the right Cauchy-Green deformation tensor, $\mathbf{R}[\mathbf{C}(\tau),t-\tau]$ is the tensorial relaxation function, and p is the Lagrange multiplier that accounts for incompressibility. The term $\mathbf{F}(t)\mathbf{R}[\mathbf{C}(t),0]$ represents the instantaneous elastic contribution to the total stress at time t.

Rat tail tendon fascicles are assumed to be transversely isotropic and incompressible. The tensorial relaxation function, $\mathbf{R}$, is selected to depend only on the fourth invariant of $\mathbf{C}$, $\mathrm{I}_4 = \mathbf{m}\cdot\mathbf{C}\mathbf{m}$, where $\mathbf{m}$ is a unit vector in the reference configuration which defines the axis of material symmetry. Therefore,

$$\mathbf{R}[I_4(\tau),t-\tau] = \left[c_1\left(e^{c_2(I_4(\tau)-1)}-1\right)\right] \left[(1-\alpha)\,e^{-(t-\tau)\beta}+\alpha\right](\mathbf{m}\otimes\mathbf{m}) \quad (2)$$

where c_1 and c_2 are non-negative constants and $\alpha = \alpha(I_4(\tau))$ and $\beta = \beta(I_4(\tau))$ are functions of I_4. In Eq. 2, the expression in the first square brackets defines the strain stiffening elastic behavior of collagenous tissues while the expression in the second square brackets defines the normalized relaxation behavior.

The fascicles are assumed to undergo a homogeneous isochoric axisymmetric deformation and have traction-free boundary conditions on their lateral surface. By using Eqs. (1) and (2) one can determine the only non-zero component of the first Piola-Kirchhoff stress tensor which defines the nominal axial stress.

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EXPERIMENTAL METHODS

Tail tendons were excised from two male Sprague Dawley rats (235 g, 236 g) immediately after death. The fascicles were stored frozen (-20°C) and before testing they were allowed to come to room temperature. Images of each fascicle were collected using a microscope (Stemi 2000C, Zeiss) and used to calculate the average cross-sectional area of each specimen assuming a circular cross-section. Black ink was sprayed on the surface of the fascicles to produce marks with suitable contrast for strain calculation.

A custom designed micro-testing device was built to perform stress relaxation tests [5]. The load was measured using a 8.9 N load cell (LSB 200, Futek) with a resolution of $\pm$ 0.02 N. The motion of the ink marks was tracked using a digital image correlation method (MATLAB v. 7.10, MathWorks) and the axial stretch was computed from the measured displacements. The nominal axial stress was computed by dividing the load by the measured cross-sectional area.

Each sample was pre-loaded to 0.1 N and then preconditioned at 6 mm/min to 0.4 mm ($\approx 0.5\%$) for 5 cycles followed by a five minute recovery period. Finally, the sample was stretched to the desired displacement for relaxation testing corresponding to 0.25 mm ($n = 3$), 0.75 mm ($n = 8$), 1.25 mm ($n = 7$) or 1.75 mm ($n = 6$) at 6 mm/min and held for 10 minutes.

RESULTS

Stress-stretch data were collected for 24 rat tail tendon fascicles. These data sets were obtained by axially stretching the fascicle along its long axis up to the displacement that was held constant during the stress relaxation experiment. The fascicles exhibited the typical nonlinear elastic strain-stiffening behavior of soft collagenous tissues. The stress-stretch data sets were used to compute the model parameters c_1 and c_2, which define the instantaneous elastic response of each fascicle.

The applied displacements (0.25 mm - 1.75 mm) produced strains in the fascicles that varied from 0.3% to 2.8%. The stress relaxation data collected at the different strain levels were used to determine the values of $\alpha = \alpha(I_4)$ and $\beta = \beta(I_4)$ in Eq. (2) at the fixed strain levels. When curve fitting the proposed model to the stress relaxation data, the values of the parameters c_1 and c_2 were fixed to those previously computed by fitting the elastic stress-stretch data. The model was fit to the experimental relaxation data by employing a nonlinear least squares algorithm implemented in MATLAB. The parameter α was constrained to be such that $0 < \alpha < 1$ and β was constrained to be non-negative.

Stress relaxation data sets for five stretch levels are shown on a log-log plot in Fig. 1 along with model fits and QLV predictions. The proposed model was able to describe the strain dependent stress relaxation behavior well ($0.80 < R^2 < 0.99$, for all 24 data sets). It can be observed that the QLV model can capture the stress relaxation response only for the first 30s of the tests but fails to describe the long time relaxation behavior.

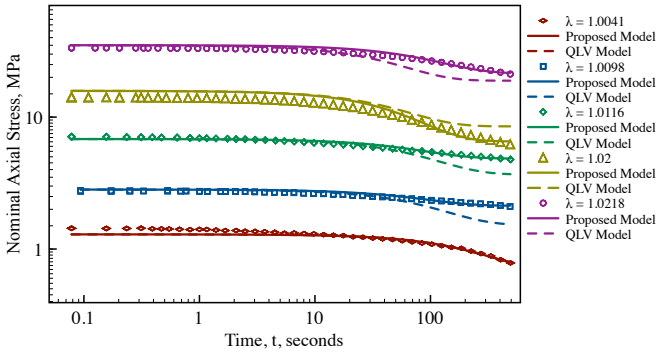

FIGURE 1: EXPERIMENTAL RELAXATION CURVES FOR RAT TAIL TENDON FASCICLES (POINTS), MODEL FIT TO DATA (SOLID LINE), AND COMPARISON TO QLV (DASHED LINE).

DISCUSSION

A constitutive relation has been formulated which can describe the nonlinear viscoelastic behavior of collagenous tissues with fibers oriented along a preferred direction, such as ligament and tendon. The constitutive model is able to capture the coupled strain and time dependent relaxation behavior with two parameters that describe the elastic behavior and two functions which describe the viscoelastic behavior. Only a few models exist in the literature to characterize the coupled strain and time dependent viscoelastic behavior of soft tissues [2]. However, no other models have been formulated which are three-dimensional, finite strain, non-separable and consider tissue anisotropy.

ACKNOWLEDGMENT

Supported by Ford Foundation Pre-Doctoral Fellowship, NSF Graduate Research Fellowship Program and NSF GARDE Program under Grant No. 0932024.

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